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Homodyne Statistics of a Vector in a Deformed Hilbert Space

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In this paper we study homodyne statistics of some vectors on a deformed Hilbert space.

1. INTRODUCTION

In a direct detection of statistics (Braunstein, 1990; Braunstein and Caves, 1990) in a single-mode photon field we count the number of photons in the field mode of interest. The probability for counting n photons, called the photon count distribution, is given by

$$P_n = \langle n|\rho|n\rangle,$$

where $|n\rangle$ is a photon-number eigenstate and ρ is a density operator. However, the direct detection cannot differentiate between the quadratures. All practical phase-sensitive measurements require a reference beam, to act as a phase reference, commonly called the local oscillator. This beam has to be phase locked to the input, otherwise it cannot provide a phase reference to distinguish between the quadratures. If the local oscillator is resonant with the system field, that is has the same frequency as the input, then this type of measurement is known as *homodyne* detection. Alternatively, if the local oscillator is detuned, that is, frequency shifted from the bandwidth of the system field, then this is known as *heterodyne* detection. It is usually assumed that the observable measured by the homodyne detector is one of the field quadratures and that by the heterodyne detector is both the field quadratures.

The Q function defined by

$$Q(\alpha) = \langle\alpha|\rho|\alpha\rangle,$$

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where $|\alpha\rangle$ is a coherent state, provides a normalized phase-space probability distribution, $Q(\alpha)/\pi$, for a quantum system. $Q(\alpha)$ can be measured in a series of optimized simultaneous measurements of two orthogonal quadrature components, because $Q(\alpha)/\pi$ gives directly the statistics of such measurements. This means that in optical frequency detection, $Q(\alpha)/\pi$ gives the statistics of heterodyne detection that measures orthogonal quadrature components of the statistics of a pair of homodyne detectors whose local oscillators have relative phases corresponding to measuring orthogonal quadrature components.

Homodyne detection is a well-established method for measuring phase-senitive properties of light. Usual process is to superimpose a signal field with a much stronger local oscillator. As a consequence the resulting field is rather strong and can be detected with photodiodes. In such a scheme a photocurrent is produced, which may be treated classically. Nevertheless, from the statistical properties of this classical current one may get some insight into the nonclassical statistics of light.

In recent years (Das, 1998, 1999a,b, 2001a,b), we have studied coherent vectors, phase vectors, coherent phase vectors, kerr vectors, and squeezed vectors in the setting of a deformed Hilbert space and plan to study here their direct, heterodyne, and homodyne statistics in this generalized setting. Here, we adopt the viewpoint of Vogel and coworker (1990, 1991) to study the statistics of different vectors so far generalized.

The work is organized as follows. In section 2, we give a brief description of preliminaries and notations. In section 3, we study the statistics of photon count. In section 4, we describe the statistics of heterodyne detection. In section 5, to study the statistics of homodyne detection we first find field strength vector and then, in section 6, through various examples we describe homodyne statistics of different vectors under consideration. Finally, we give a conclusion.

2. PRELIMINARIES AND NOTATIONS

We consider the set

$$H_q = \left\{ f : f(z) = \sum a_n z^n \quad \text{where} \quad \sum [n]!\, |a_n|^2 < \infty \right\},$$

where $[n] = \frac{1-q^n}{1-q}$, $0 < q < 1$.

For $f, g \in H_q$, $f(z) = \sum_{n=0}^{\infty} a_n z^n$, $g(z) = \sum_{n=0}^{\infty} b_n z^n$ we define addition and scalar multiplication as follows:

$$(f + g)(z) = f(z) + g(z) = \sum_{n=0}^{\infty} (a_n + b_n) z^n \tag{1}$$

and

$$(\lambda \circ f)(z) = \lambda \circ f(z) = \sum_{n=0}^{\infty} \lambda a_n z^n. \tag{2}$$

It is easily seen that H_q forms a vector space with respect to usual pointwise scalar multiplication and pointwise addition by (1) and (2). We observe that $e_q(z) = \sum_{n=0}^{\infty} \frac{z^n}{[n]!}$ belongs to H_q.

Now we define the inner product of two functions $f(z) = \sum a_n z^n$ and $g(z) = \sum b_n z^n$ belonging to H_q as

$$(f, g) = \sum [n]!\, \bar{a}_n b_n. \tag{3}$$

Corresponding norm is given by

$$\| f \|^2 = (f, f) = \sum [n]! |a_n|^2 < \infty.$$

With this norm derived from the inner product it can be shown that H_q is a complete normed space. Hence H_q forms a Hilbert space.

In a recent paper (Das, 1998, 1999a) we have proved that the set $\{\frac{z^n}{\sqrt{[n]!}}, n = 0, 1, 2, 3, \ldots\}$ forms a complete orthonormal set. If we consider the following actions on H_q

$$\begin{aligned} T f_n &= \sqrt{[n]}\, f_{n-1}, \\ T^* f_n &= \sqrt{[n+1]}\, f_{n+1}, \end{aligned} \tag{4}$$

where T is the backward-shift and its adjoint T^* is the forward-shift operator on H_q and $f_n(z) = \frac{z^n}{\sqrt{[n]!}}$ then we have shown (Das, 1998, 1999a) that the solution of the following eigenvalue equation

$$T f_\alpha = \alpha f_\alpha \tag{5}$$

is given by

$$f_\alpha = e_q(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{[n]!}} f_n. \tag{6}$$

We call f_α a **coherent vector** in H_q.

3. STATISTICS OF PHOTON COUNT

In this section we shall describe statistics of direct detection, that is, the probability distribution P_n of different vectors under consideration.

3.1. Incoherent Vectors

For the incoherent vectors (Das, 2001b) we take the density operator to be

$$\rho = \sum_{n=0}^{\infty} p_n | f_n\rangle\langle f_n|, \tag{7}$$

with

$$p_n \geq 0 \quad \text{and} \quad \sum_{n=0}^{\infty} p_n = 1.$$

Then we calculate photon count distribution P_n as

$$P_n = (f_n, \rho f_n) = \sum_{m=0}^{\infty} p_m (f_n, f_m)(f_m, f_n) = p_n.$$

3.2. Coherent Vector

For the coherent vectors f_α (Das, 1998),

$$f_\alpha = e_q(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{[n]!}} f_n. \tag{8}$$

We take the density operator to be

$$\rho = | f_\alpha\rangle\langle f_\alpha|, \qquad \alpha = |\alpha| e^{i\theta_0} \tag{9}$$

and calculate the photon count distribution P_n as

$$\begin{aligned} P_n &= (f_n, \rho f_n) \\ &= (f_n, | f_\alpha\rangle\langle f_\alpha | f_n) \\ &= |(f_n, f_\alpha)|^2 \\ &= e_q(|\alpha|^2)^{-1} \frac{|\alpha|^{2n}}{[n]!}. \end{aligned} \tag{10}$$

3.3. Coherent Phase Vector

For a coherent phase vector f_β (Das, 1999b),

$$f_\beta = \Phi(|\beta|^2)^{-1/2} \sum_{n=0}^{\infty} \beta^n \sqrt{\frac{(q + [0]) \cdots (q^n + [n-1])}{[n]!}} f_n, \tag{11}$$

with $|\beta| < 1$ and

$$\Phi(|\beta|^2) = \sum_{n=0}^{\infty} |\beta|^{2n} \frac{(q + [0]) \cdots (q^n + [n-1])}{[n]!}. \tag{12}$$

We take the density operator to be

$$\rho = |f_\beta\rangle\langle f_\beta| \tag{13}$$

and calculate the photon count distribution $P(n)$ as

$$\begin{aligned} P_n &= (f_n, \rho f_n) \\ &= (f_n, |f_\beta\rangle\langle f_\beta| f_n) \\ &= |(f_n, f_\beta)|^2 \\ &= \Phi(|\beta|^2)^{-1} |\beta|^{2n} \frac{(q + [0]) \cdots (q^n + [n-1])}{[n]!}. \end{aligned} \tag{14}$$

3.4. Kerr Vector

For a kerr vector ϕ_α^K (Das, 1999b),

$$\begin{aligned} \phi_\alpha^K &= e_q^{\frac{i}{2}\gamma N(N-1)} f_\alpha \\ &= \sum_{n=0}^{\infty} k_n f_n, \end{aligned} \tag{15}$$

where

$$k_n = e_q(|\alpha|^2)^{-1/2} \frac{\alpha^n}{\sqrt{[n]!}} e_q^{\frac{i}{2}\gamma [n]([n]-1)}. \tag{16}$$

We take the density operator to be

$$\rho = \left|\phi_\alpha^K\right\rangle\!\left\langle\phi_\alpha^K\right| \tag{17}$$

and calculate the photon count distribution $P(n)$ as

$$\begin{aligned} P(n) &= (f_n, \rho f_n) \\ &= \left(f_n, \left|\phi_\alpha^K\right\rangle\!\left\langle\phi_\alpha^K\right| f_n\right) \\ &= \left|\left(f_n, \phi_\alpha^K\right)\right|^2 \\ &= e_q(|\alpha|^2)^{-1} \frac{|\alpha|^{2n}}{[n]!} \left| e_q^{\frac{i}{2}\gamma [n]([n]-1)} \right|^2. \end{aligned} \tag{18}$$

3.5. Squeezed Vector

For a squeezed vector f_s (Das, 2001a),

$$f_s = \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \sum_{n=0}^{\infty} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}} f_{2n}. \tag{19}$$

We take the density operator to be

$$\rho = | f_s\rangle\langle f_s | \tag{20}$$

and calculate the photon count distribution $P(n)$ as

$$\begin{aligned} P(n) &= (f_n, \rho f_n) \\ &= (f_n, f_s\rangle\langle f_s | f_n) \\ &= |(f_n, f_s)|^2 \\ &= \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1} |\alpha|^n \frac{[n-1]!!}{[n]!!}. \end{aligned} \tag{21}$$

4. STATISTICS OF HETERODYNE DETECTION

In this section we shall describe statistics of heterodyne detection, that is, the probability distribution $Q(\alpha)/\pi$ of different vectors under consideration.

4.1. Incoherent Vector

For the incoherent vectors (Das, 2001b) we take the density operator to be

$$\rho = \sum_{n=0}^{\infty} p_n | f_n\rangle\langle f_n |, \tag{22}$$

with

$$p_n \geq 0 \quad \text{and} \quad \sum_{n=0}^{\infty} p_n = 1.$$

Then we calculate phase-space probability distribution $Q(\alpha)/\pi$ as

$$\begin{aligned} Q(\alpha)/\pi &= 1/\pi (f_\alpha, \rho f_\alpha) \\ &= 1/\pi \sum_{n=0}^{\infty} p_n |(f_n, f_\alpha)|^2 \\ &= 1/\pi\, e_q(|\alpha|^2)^{-1} \sum_{n=0}^{\infty} p_n \frac{|\alpha|^{2n}}{[n]!}. \end{aligned} \tag{23}$$

4.2. Coherent Vector

For the coherent vectors $f_{\alpha'}$ (Das, 1998),

$$f_{\alpha'} = e_q(|\alpha'|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha'^n}{\sqrt{[n]!}} f_n. \tag{24}$$

We take the density operator to be

$$\rho = |f_{\alpha'}\rangle\langle f_{\alpha'}|, \alpha' = |\alpha'|e^{i\theta_0} \tag{25}$$

and calculate the phase-space probability distribution $Q(\alpha)/\pi$ as

$$\begin{aligned} Q(\alpha)/\pi &= 1/\pi(f_\alpha, \rho f_\alpha) \\ &= 1/\pi(f_\alpha, |f_{\alpha'}\rangle\langle f_{\alpha'}| f_\alpha) \\ &= 1/\pi|(f_\alpha, f_{\alpha'})|^2 \\ &= 1/\pi\, e_q(|\alpha|^2)^{-1} e_q(|\alpha'|^2)^{-1} |e_q(\bar{\alpha}\alpha')|^2. \end{aligned} \tag{26}$$

4.3. Coherent Phase Vector

For a coherent phase vector f_β (Das, 1999b),

$$f_\beta = \Phi(|\beta|^2)^{-1/2} \sum_{n=0}^{\infty} \beta^n \sqrt{\frac{(q+[0])\cdots(q^n+[n-1])}{[n]!}} f_n, \tag{27}$$

with $|\beta| < 1$ and

$$\Phi(|\beta|^2) = \sum_{n=0}^{\infty} |\beta|^{2n} \frac{(q+[0])\cdots(q^n+[n-1])}{[n]!}. \tag{28}$$

We take the density operator to be

$$\rho = |f_\beta\rangle\langle f_\beta| \tag{29}$$

and calculate the phase-space probability distribution $Q(\alpha)/\pi$ as

$$\begin{aligned} Q(\alpha)/\pi &= 1/\pi(f_\alpha, \rho f_\alpha) \\ &= 1/\pi(f_\alpha, |f_\beta\rangle\langle f_\beta| f_\alpha) \\ &= 1/\pi|(f_\alpha, f_\beta)|^2 \\ &= 1/\pi\, \Phi(|\beta|^2)^{-1} e_q(|\alpha|^2)^{-1} |\Phi(\bar{\alpha}\beta)|^2. \end{aligned} \tag{30}$$

4.4. Kerr Vector

For a kerr vector ϕ_α^K (Das, 1999b),

$$\begin{aligned}\phi_\alpha^K &= e_q^{\frac{i}{2}\gamma N(N-1)} f_\alpha \\ &= \sum_{n=0}^{\infty} k_n f_n,\end{aligned} \tag{31}$$

where

$$k_n = e_q(|\alpha|^2)^{-1/2} \frac{\alpha^n}{\sqrt{[n]!}} e_q^{\frac{i}{2}\gamma[n]([n]-1)}. \tag{32}$$

We take the density operator to be

$$\rho = \left|\phi_\alpha^K\right\rangle\!\left\langle\phi_\alpha^K\right| \tag{33}$$

and calculate the phase-space probability distribution $Q(\alpha)/\pi$ as

$$\begin{aligned}Q(\alpha)/\pi &= 1/\pi(f_\alpha, \rho f_\alpha) \\ &= 1/\pi\left(f_\alpha, \left|\phi_\alpha^K\right\rangle\!\left\langle\phi_\alpha^K\right| f_\alpha\right) \\ &= 1/\pi\left|\left(f_\alpha, \phi_\alpha^K\right)\right|^2 \\ &= 1/\pi\, e_q(|\alpha|^2)^{-2}\left|\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{[n]!} e_q^{\frac{i}{2}\gamma[n]([n]-1)}\right|^2.\end{aligned} \tag{34}$$

4.5. Squeezed Vector

For a squeezed vector f_s (Das, 2001a),

$$f_s = \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2} \sum_{n=0}^{\infty} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}}\, f_{2n}. \tag{35}$$

We take the density operator to be

$$\rho = |f_s\rangle\langle f_s| \tag{36}$$

and calculate the phase-space probability distribution $Q(\alpha)/\pi$ as

$$\begin{aligned}Q(\alpha)/\pi &= 1/\pi(f_\alpha, \rho f_\alpha) \\ &= 1/\pi(f_{\{\alpha\}},\ f_{\{s\}}\rangle\langle f_{\{s\}}|\, f_\alpha) \\ &= 1/\pi\,|(f_\alpha,\ f_s)|^2\end{aligned}$$

$$= 1/\pi\, e_q(|\alpha|^2)^{-1}\left[\sum_{n=0}^{\infty}|\alpha|^{2n}\frac{[2n-1]!!}{[2n]!!}\right]^{-1}$$

$$\times\left|\sum_{n=0}^{\infty}\frac{\overline{\alpha}^{2n}}{\sqrt{[2n]!}}\alpha^n\sqrt{\frac{[2n-1]!!}{[2n]!!}}\right|^2. \tag{37}$$

5. STATISTICS OF HOMODYNE DETECTION

Homodyne detection measures a quadrature component which is the field operator

$$\hat{E}(\phi) = e^{-i\phi}T - e^{i\phi}T^*,$$

where ϕ is a phase determined by the phase of the local oscillator. The statistics of homodyne detection, that is, the probability distribution $P(E(\phi))$ of the quadrature component is given by

$$P(E(\phi)) = (E(\phi), \rho E(\phi)),$$

where $E(\phi)$ denotes an arbitrary vector satisfying the equation

$$(e^{-i\phi}T - e^{i\phi}T^*)E(\phi) = 0$$

and ρ is a density operator. The operators T and T^* are elaborated in section 2. It is called a field-strength vector $E(\phi)$. We shall find the probability distribution at the particular field value $E(\phi) = 0$. Before we proceed to find the homodyne statistics of various vectors under consideration we find the field-strength vector at the origin $E(\phi) = 0$.

5.1. Generation of Field-Strength Vector

The field-strength vector at the origin $E(\phi) = 0$ is generated by the action of $e^{-i\phi}T - e^{i\phi}T^*$ on an arbitrary vector f_β in H_q (Das, 1998), which satisfies the following equation

$$(e^{-i\phi}T - e^{i\phi}T^*)\, f_\beta = 0, \tag{38}$$

where

$$f_\beta(z) = \sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} a_n\sqrt{[n]!}\, f_n(z), \tag{39}$$

or

$$f_\beta = \sum_{n=0}^{\infty} a_n\sqrt{[n]!}\, f_n.$$

We have

$$e^{-i\phi}Tf_\beta = \sum_{n=0}^{\infty} e^{-i\phi}a_n\sqrt{[n]!}\,Tf_n$$
$$= \sum_{n=1}^{\infty} e^{-i\phi}a_n\sqrt{[n]!}\sqrt{[n]}\,f_{n-1}$$
$$= \sum_{n=0}^{\infty} e^{-i\phi}a_{n+1}\sqrt{[n+1]!}\sqrt{[n+1]}\,f_n \tag{40}$$

and

$$e^{i\phi}T^*f_\beta = \sum_{n=0}^{\infty} e^{i\phi}a_n\sqrt{[n]!}\,T^*f_n$$
$$= \sum_{n=0}^{\infty} e^{i\phi}a_n\sqrt{[n]!}\sqrt{[n+1]}\,f_{n+1}. \tag{41}$$

Now from (38)–(41) we observe that a_n satisfies the following difference equation:

$$e^{-i\phi}a_{n+1}\sqrt{[n+1]!}\sqrt{[n+1]} = e^{i\phi}a_{n-1}\sqrt{[n-1]!}\sqrt{[n]} \tag{42}$$

That is,

$$a_{n+2} = e^{2i\phi}\frac{\sqrt{[n]!}}{\sqrt{[n+2]!}}\frac{\sqrt{[n+1]}}{\sqrt{[n+2]}}a_n \tag{43}$$

and

$$a_1 = 0. \tag{44}$$

Hence,

$$a_2 = e^{2i\phi}\frac{\sqrt{[0]!}}{\sqrt{[2]!}}\frac{\sqrt{[1]}}{\sqrt{[2]}}a_0$$
$$a_4 = e^{2i\phi}\frac{\sqrt{[2]!}}{\sqrt{[4]!}}\frac{\sqrt{[3]}}{\sqrt{[4]}}a_2 = e^{4i\phi}\frac{\sqrt{[2]!}\sqrt{[0]!}}{\sqrt{[4]!}\sqrt{[2]!}}\frac{\sqrt{[3]}\sqrt{[1]}}{\sqrt{[4]}\sqrt{[2]}}a_0$$
$$a_6 = e^{2i\phi}\frac{\sqrt{[4]!}}{\sqrt{[6]!}}\frac{\sqrt{[5]}}{\sqrt{[6]}}a_4 = e^{6i\phi}\frac{\sqrt{[4]!}\sqrt{[2]!}\sqrt{[0]!}}{\sqrt{[6]!}\sqrt{[4]!}\sqrt{[2]!}}\frac{\sqrt{[5]}\sqrt{[3]}\sqrt{[1]}}{\sqrt{[6]}\sqrt{[4]}\sqrt{[2]}}a_0$$

and so on. Thus,

$$a_{2n} = e^{n(2i\phi)}\frac{1}{\sqrt{[2n]!}}\frac{[2n-1]!!}{\sqrt{[2n]!!}}a_0$$

and

$$a_1 = a_3 = a_5 = \cdots = a_{2n-1} = 0.$$

Thus, f_β satisfying (38) has the form

$$f_\beta = \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n = a_0 \sum_{n=0}^{\infty} e^{n(2i\phi)} \sqrt{\frac{[2n-1]!!}{[2n]!!}}\, f_{2n}. \tag{45}$$

To normalise we have

$$1 = (f_\beta,\ f_\beta) = |a_0|^2 \sum_{n=0}^{\infty} \frac{[2n-1]!!}{[2n]!!}. \tag{46}$$

Thus, aside from a trivial phase we have

$$a_0 = \left[\sum_{n=0}^{\infty} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \tag{47}$$

and the field-strength vector at the origin f_β takes the form

$$f_\beta = \left[\sum_{n=0}^{\infty} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \sum_{n=0}^{\infty} e^{n(2i\phi)} \sqrt{\frac{[2n-1]!!}{[2n]!!}}\, f_{2n}. \tag{48}$$

Henceforth, we shall denote this vector as

$$(E(\phi) = 0) = \left[\sum_{n=0}^{\infty} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \sum_{n=0}^{\infty} e^{n(2i\phi)} \sqrt{\frac{[2n-1]!!}{[2n]!!}}\, f_{2n}. \tag{49}$$

6. EXAMPLES

We now calculate homodyne statistics for various vectors under consideration.

6.1. Incoherent Vector

For the incoherent vectors (Das, 2001b) we take the density operator to be

$$\rho = \sum_{n=0}^{\infty} p_n |\, f_n\rangle\langle\, f_n|, \tag{50}$$

with

$$p_n \geq 0 \quad \text{and} \quad \sum_{n=0}^{\infty} p_n = 1.$$

Then we calculate homodyne distribution $P(E(\phi)=0)$ as

$$\begin{aligned} P(E(\phi)=0) &= (E(\phi)=0,\, \rho E(\phi)=0)\\ &= \sum_{n=0}^{\infty} p_n |(E(\phi)=0,\ f_n)|^2\\ &= \sum_{n=0}^{\infty} p_n \left[\sum_{n=0}^{\infty} \frac{[2n-1]!!}{[2n]!!}\right]^{-1} \left|\sum_{n=0}^{\infty} e^{in\phi} \sqrt{\frac{[n-1]!!}{[n]!!}}\right|^2. \end{aligned} \tag{51}$$

6.2. Coherent Vector

For the coherent vectors f_α (Das, 1998),

$$f_\alpha = e_q(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{[n]!}} f_n. \tag{52}$$

We take the density operator to be

$$\rho = |f_\alpha\rangle\langle f_\alpha|, \qquad \alpha = |\alpha| e^{i\theta_0} \tag{53}$$

and calculate homodyne distribution $P(E(\phi)=0)$ as

$$\begin{aligned} P(E(\phi)=0) &= (E(\phi)=0,\, \rho E\,(\phi)=0)\\ &= (E(\phi)=0,\, |f_\alpha\rangle\langle f_\alpha| E(\phi)=0)\\ &= |(E(\phi)=0,\ f_\alpha)|^2\\ &= e_q(|\alpha|^2)^{-1} \left[\sum_{n=0}^{\infty} \frac{[2n-1]!!}{[2n]!!}\right]^{-1}\\ &\quad \times \left|\sum_{n=0}^{\infty} e^{in\phi} \sqrt{\frac{[n-1]!!}{[n]!!}} \frac{\alpha^n}{\sqrt{[n]!}}\right|^2. \end{aligned} \tag{54}$$

6.3. Coherent Phase Vector

For a coherent phase vector f_β (Das, 1999b),

$$f_\beta = \Phi(|\beta|^2)^{-1/2} \sum_{n=0}^{\infty} \beta^n \sqrt{\frac{(q+[0])\cdots(q^n+[n-1])}{[n]!}}\, f_n \tag{55}$$

with $|\beta| < 1$ and

$$\Phi(|\beta|^2) = \sum_{n=0}^{\infty} |\beta|^{2n} \frac{(q+[0])\cdots(q^n+[n-1])}{[n]!}. \tag{56}$$

We take the density operator to be

$$\rho = |f_\beta\rangle\langle f_\beta| \tag{57}$$

and calculate homodyne distribution $P(E(\phi) = 0)$ as

$$\begin{aligned} P(E(\phi) = 0) &= (E(\phi) = 0, \rho E(\phi) = 0) \\ &= (E(\phi) = 0, |f_\beta\rangle\langle f_\beta|E(\phi) = 0) \\ &= |(E(\phi) = 0,\ f_\beta)|^2 \\ &= \Phi(|\beta|^2)^{-1}\left[\sum_{n=0}^{\infty}\frac{[2n-1]!!}{[2n]!!}\right]^{-1}\left|\sum_{n=0}^{\infty} e^{in\phi}\sqrt{\frac{[n-1]!!}{[n]!!}}\beta^n \right. \\ &\quad \left. \times\sqrt{\frac{(q+[0])\cdots(q^n+[n-1])}{[n]!}}\right|^2. \end{aligned} \tag{58}$$

6.4. Kerr Vector

For a kerr vector ϕ_α^K (Das, 1999b),

$$\begin{aligned} \phi_\alpha^K &= e_q^{\frac{i}{2}\gamma N(N-1)} f_\alpha \\ &= \sum_{n=0}^{\infty} k_n f_n, \end{aligned} \tag{59}$$

where

$$k_n = e_q(|\alpha|^2)^{-1/2}\frac{\alpha^n}{\sqrt{[n]!}} e_q^{\frac{i}{2}\gamma[n]([n]-1)}. \tag{60}$$

We take the density operator to be

$$\rho = \left|\phi_\alpha^K\right\rangle\!\left\langle\phi_\alpha^K\right| \tag{61}$$

and calculate homodyne distribution $P(E(\phi) = 0)$ as

$$\begin{aligned} P(E(\phi) = 0) &= (E(\phi) = 0, \rho E(\phi) = 0) \\ &= (E(\phi) = 0, \left|\phi_\alpha^K\right\rangle\!\left\langle\phi_\alpha^K\right| E(\phi) = 0) \\ &= \left|\left(E(\phi) = 0, \phi_\alpha^K\right)\right|^2 \\ &= \left[\sum_{n=0}^{\infty}\frac{[2n-1]!!}{[2n]!!}\right]^{-1}\left|\sum_{n=0}^{\infty} e^{in\phi}\sqrt{\frac{[n-1]!!}{[n]!!}}k_n\right|^2. \end{aligned} \tag{62}$$

6.5. Squeezed Vector

For a squeezed vector f_s (Das, 2001a),

$$f_s = \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \sum_{n=0}^{\infty} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}} f_{2n}. \tag{63}$$

We take the density operator to be

$$\rho = | f_s \rangle \langle f_s | \tag{64}$$

and calculate homodyne distribution $P(E(\phi) = 0)$ as

$$\begin{aligned} P(E(\phi) = 0) &= (E(\phi) = 0, \rho E(\phi) = 0) \\ &= (E(\phi) = 0, f_s \rangle \langle f_s | E(\phi) = 0) \\ &= |(E(\phi) = 0, f_s|^2 \\ &= \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1} \left[\sum_{n=0}^{\infty} \frac{[2n-1]!!}{[2n]!!} \right]^{-1} \\ &\quad \times \left| \sum_{n=0}^{\infty} e^{2in\phi} \alpha^n \frac{[2n-1]!!}{[2n]!!} \right|^2 . \end{aligned} \tag{65}$$

7. CONCLUSION

In conclusion, we have studied the statistics of direct, heterodyne, and homodyne detection for several vectors under consideration. Using field-strength eigenvectors we have given a prescription for the measurement of the distribution using a balanced homodyne detection scheme in the deformed case.

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